# Channel-Dependent Session Types

**Ryan Kavanagh**, Brigitte Pientka

NJ Programming Languages and Systems Seminar, May 2022

McGill University

2022-05-19

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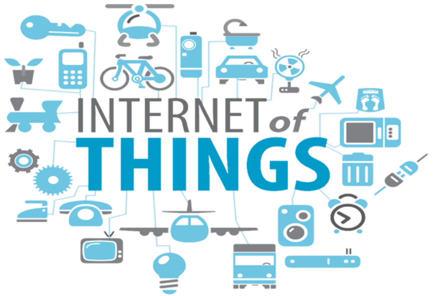
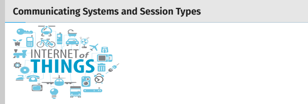
# Communicating Systems and Session Types








1. Communicating systems are ubiquitous
2. To work, every component must communicate with the others according to rules called protocols
3. Failure to do so can lead to vulnerabilities like Heartbleed
4. Caused by failure to implement TLS Heartbeat protocol extension.
5. Estimated cost to industry: over $500 million
6. Session-typed languages can help
7. Analogous to data types, but for communication
8. Today's talk: How to capture more expressive protocols

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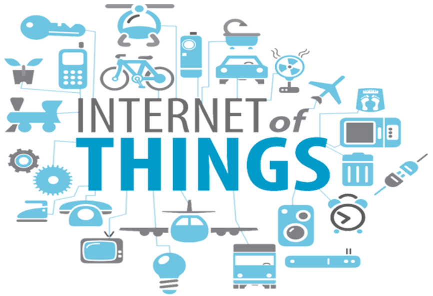







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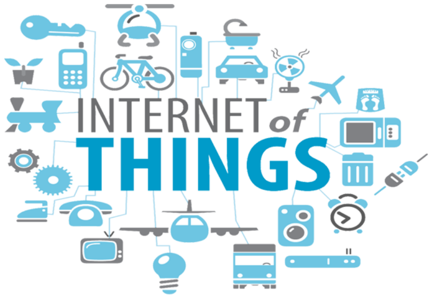


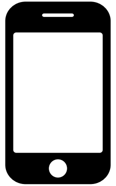






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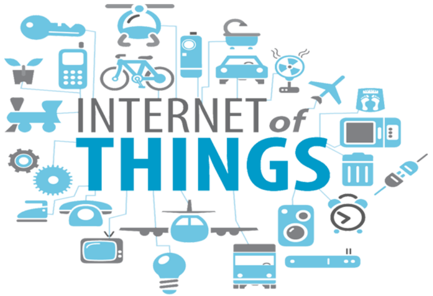


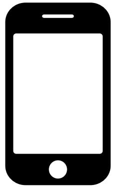





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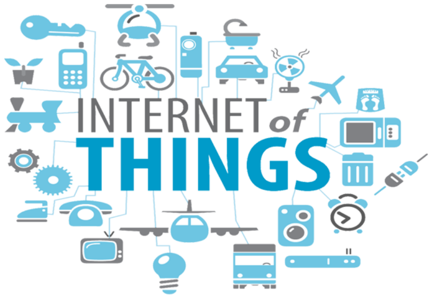
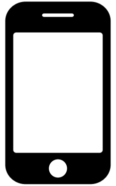
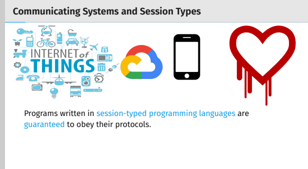


Programs written in session-typed programming languages are guaranteed to obey their protocols.







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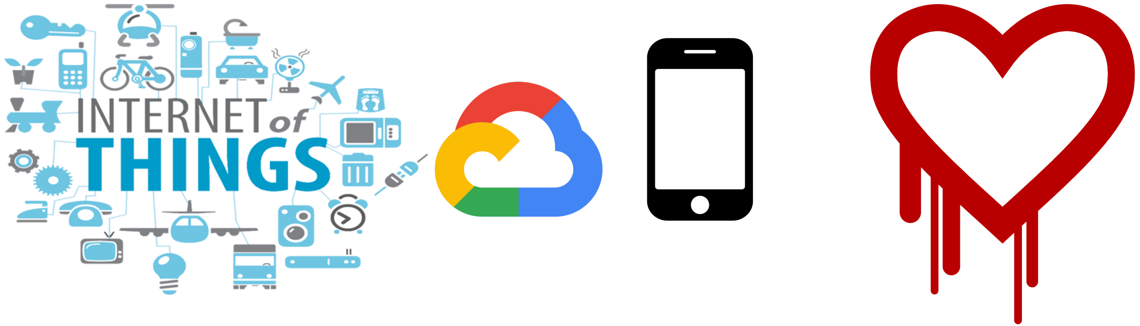


Programs written in session-typed programming languages are guaranteed to obey their protocols.

**Today:** How can we capture more expressive protocols?






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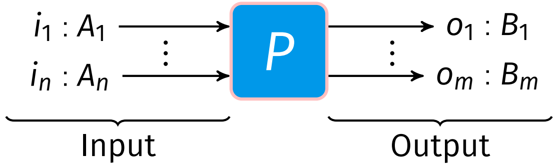
# Processes and Session-Typed Channels

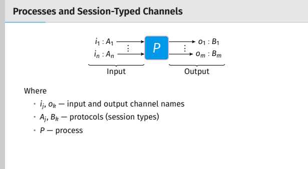



Where

- $i_j$, $o_k$ — input and output channel names
- $A_j$, $B_k$ — protocols (session types)
- $P$ — process

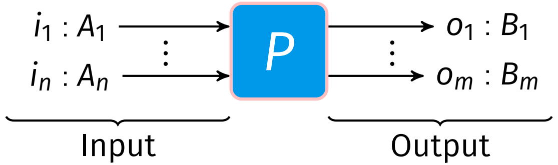






1. Think process as black boxes communicating over wires.
2. Wires are called “channels”; communication should respect a protocol.
3. The protocol specifies what kind of message can be transmitted next.
4. Protocols evolve over the course of communication to allow for different kinds of messages.
5. Make clear that channels and protocols are different.
6. In general, communication is bidirectional, but today, assume left to right.

# Processes and Session-Typed Channels



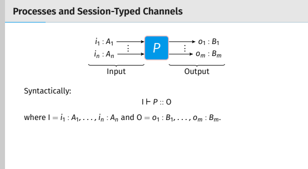

Syntactically:

$$\mathrm{I} \vdash P :: \mathrm{O}$$

where $\mathrm{I} = i_1 : A_1, \ldots, i_n : A_n$ and $\mathrm{O} = o_1 : B_1, \ldots, o_m : B_m$.







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# Bit Streams

Bit stream protocol:

$$\text{bits} = (\text{b0} \fatsemi \text{bits}) \oplus (\text{b1} \fatsemi \text{bits})$$

Example communications satisfying `bits`:

$$\xrightarrow{\text{b0 b1 b0 b0}\cdots} c : \text{bits}$$





Bit Streams
Bit stream protocol:
bits = (b0 ⨟ bits) ⊕ (b1 ⨟ bits)
Example communications satisfying bits:
b0 b1 b0 b0 ··· c : bits

1. Recurring example throughout this talk — bit streams
2. We can also deal with more interesting features like queues and stacks or channel transmission, but bit streams are useful for illustrating key features.
3. Protocol specifies what communications can be sent on a channel.
4. A communication is a sequence of messages.
5. This is a recursive protocol.
6. Send a bit, and then say that the remainder of the communication will follow the bits protocol: protocols change

# Flipping Bits


```
i : bits ⊢ F :: o : bits
F(i; o) = case i { b0 ⇒ send b1 on o; F(i; o)
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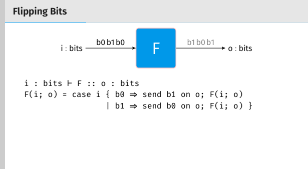

1. The bit flipping process F uses the input channel *i* that satisfies bits, and provides an output channel *o* that satisfies bits.
2. *i* and *o* are channel names; bits is the protocol
3. We can think of the typing judgment as a spec for *F*.
4. The typing judgment isn't very precise: the identity function satisfies the same specification.
5. EMPHASIZE MULTITUDE OF DIFFERENT PROCESSES
6. Want to make typing judgments capture more precise invariants relating input and output.
7. Treat session types as processes that can observe communications to produce more precise specifications.

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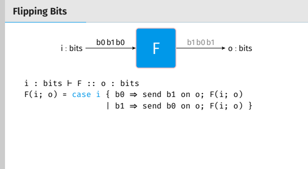

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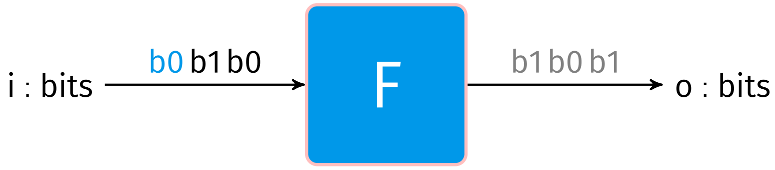


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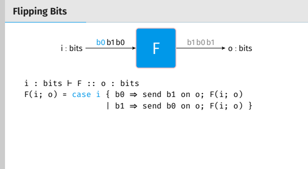

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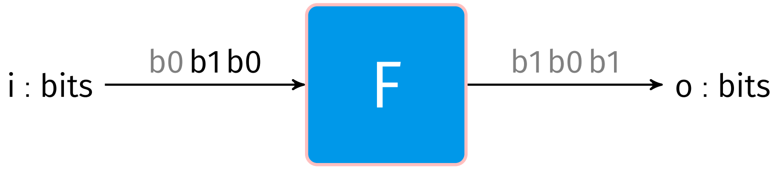


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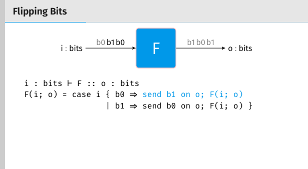

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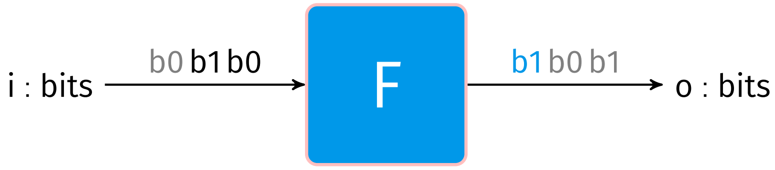


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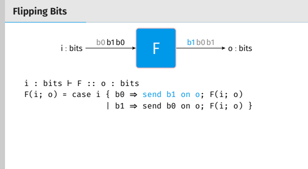

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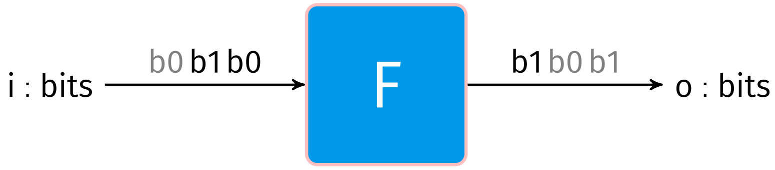


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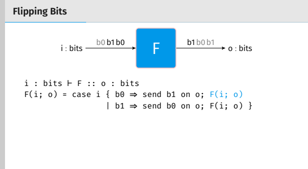

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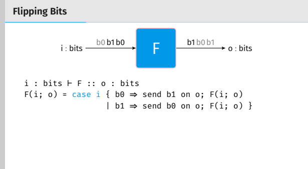


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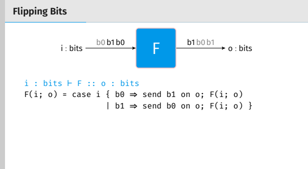

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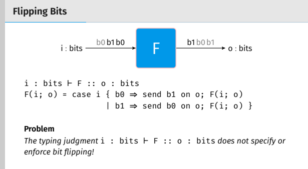
**Problem**

*The typing judgment* `i : bits ⊢ F :: o : bits` *does not specify or enforce bit flipping!*







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# Extending Session Types With Observing Processes

| | |
|---|---|
| $A, B \coloneqq \cdots$ | other session types |
| $\mid (l \mathbin{⨾} A) \oplus (r \mathbin{⨾} B)$ | labelled choice |





Extending Session Types With Observing Processes
$A, B \coloneqq \cdots$ other session types
$\mid (l \mathbin{⨾} A) \oplus (r \mathbin{⨾} B)$ labelled choice

1. Only giving syntax for binary choice, but assume it for any finite arity
2. Use definitional equality to capture type-level computation

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Extending Session Types With Observing Processes
A, B ≔ ··· other session types
| (l ⨟ A) ⊕ (r ⨟ B) labelled choice
| CASE c {l ⇒ A | r ⇒ B} label observation

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$$
\begin{aligned}
A, B &\coloneqq \cdots && \text{other session types} \\
&\mid (l \fatsemi A) \oplus (r \fatsemi B) && \text{labelled choice} \\
&\mid \text{CASE } c\ \{l \Rightarrow A \mid r \Rightarrow B\} && \text{label observation}
\end{aligned}
$$

**Operational Intuition**

CASE $c$ $\{l \Rightarrow A \mid r \Rightarrow B\} \equiv A$ if $l$ observed on channel $c$

CASE $c$ $\{l \Rightarrow A \mid r \Rightarrow B\} \equiv B$ if $r$ observed on channel $c$





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# Revisiting Bit Flipping

Bit stream protocol:

$$\text{bits} = (\text{b0} \fatsemi \text{bits}) \oplus (\text{b1} \fatsemi \text{bits})$$





Revisiting Bit Flipping
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Bit flipping protocol, assuming $i : \text{bits}$:

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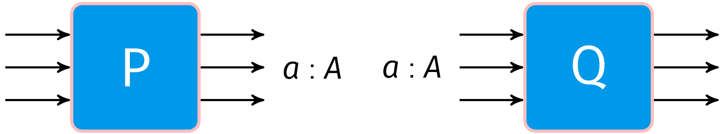






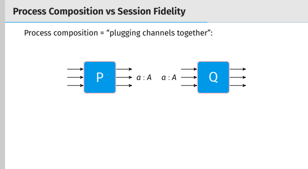

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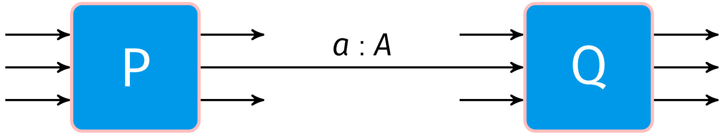






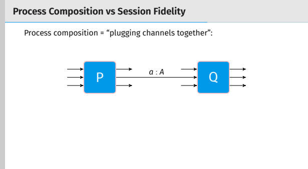


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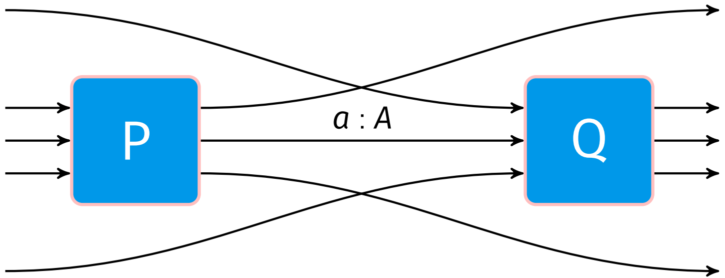






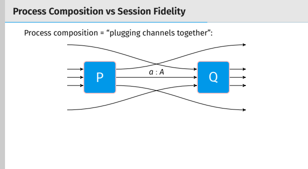


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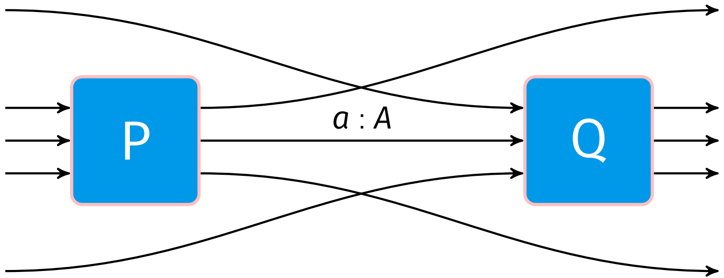
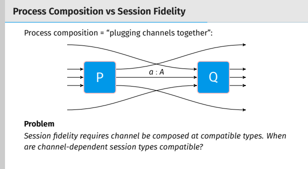



**Problem**

*Session fidelity requires channel be composed at compatible types. When are channel-dependent session types compatible?*





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3. Provides a dependency free upper bound on what $A$ allows.
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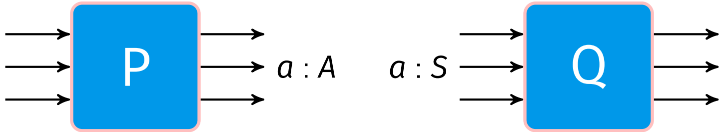






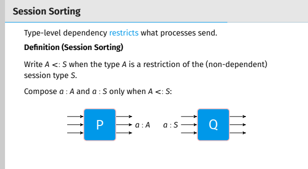

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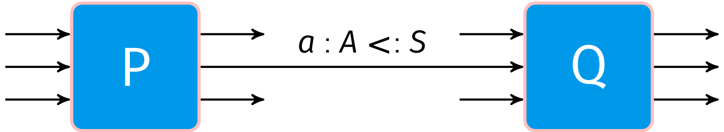






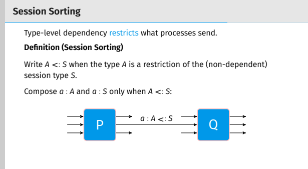

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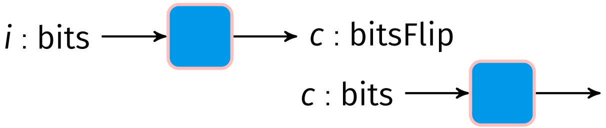






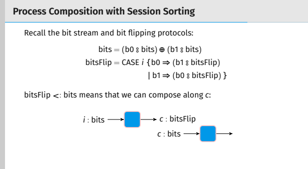

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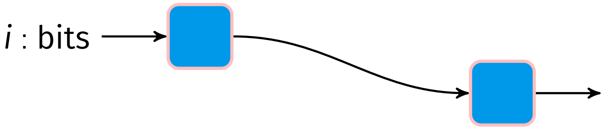






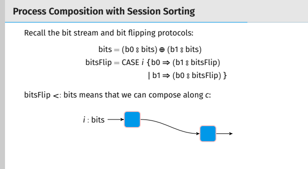


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1. VDST: depend on values from same channel. Invariants captured by sending proof terms. T/C/P 2011
2. LDST: TV19. Treat labels as first class objects. Types do a case analysis on labels sent on same channel.
3. LDST: Original motivation was to disentangle communication from introducing and eliminating values.
4. VDST/LDST: dependency only on same channel.
5. MPST: can globally specify interactions. Very rich but very complex. Typically closed world, hard to extend with new processes.
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# Future Work

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- Integrate with other forms of dependency like value- and label-dependency
- Find a logical interpretation
- Prove subject reduction
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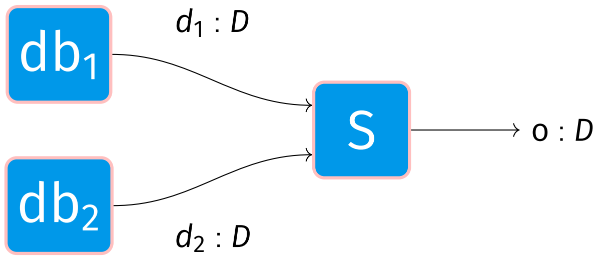
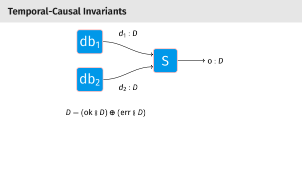
# Temporal-Causal Invariants




$$D = (\mathsf{ok} \mathbin{⅋} D) \oplus (\mathsf{err} \mathbin{⅋} D)$$





1. Backup slide
2. The bit flipping example captures information flow.
3. We can also use type-level computation to describe temporal and causal invariants.
4. Want to observe ok on $o$ only if both databases successfully committed their data.
5. Particularly useful in bidirectional settings where we can delegate communication: lets us specify how our delegates communicate.

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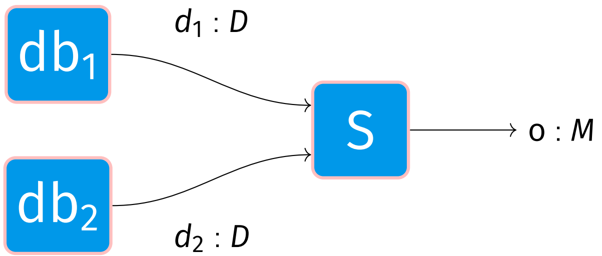


$$
\begin{aligned}
D &= (\mathsf{ok} \fatsemi D) \oplus (\mathsf{err} \fatsemi D) \\
M &= \mathsf{CASE}\ d_1\ \{\mathsf{ok} \Rightarrow \mathsf{CASE}\ d_2\ \{\mathsf{ok} \Rightarrow (\mathsf{ok} \fatsemi M) \\
&\qquad\qquad\qquad\qquad\qquad\quad \mid \mathsf{err} \Rightarrow (\mathsf{err} \fatsemi M)\} \\
&\qquad\qquad\quad \mid \mathsf{err} \Rightarrow \mathsf{CASE}\ d_2\ \{\mathsf{ok} \Rightarrow (\mathsf{err} \fatsemi M) \\
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\end{aligned}
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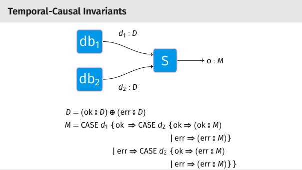

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