Some Properties of Parametrized Fixed Points on O-Categories

And Applications to Session Types

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Session-Typing Communication

C S
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A — communication protocol (session type)
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- external choice (A & B)
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- internal choice ($A \oplus B$)
- external choice ($A \& B$)
- channel transmission ($A \otimes B, A \rightarrow B$)
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\[ [A] \] — domain of (bidirectional) communications satisfying A

Want an embedding \( [A] \rightarrow [A] \times [A] \).
Open Session-Types

Generalize $[A] \rightarrow [A] \times [A]$ to open session types $\Xi \vdash A$:

$$DCPO^\Xi = \prod_{\alpha \in \Xi} DCPO$$

$[\Xi \vdash A], [\Xi \vdash A], [\Xi \vdash A] : DCPO^\Xi \xrightarrow{\text{l.c.}} DCPO$
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The embedding becomes a natural transformation:

$$[\Xi \vdash A] : [\Xi \vdash A] \Rightarrow [\Xi \vdash A] \times [\Xi \vdash A]$$

where each component of $[\Xi \vdash A]$ is an embedding.
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\[
\Xi, \alpha \vdash A \\
\Xi \vdash \text{rec}(\alpha.A)
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Recursive Session Types

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\frac{\Xi, \alpha \vdash A}{\Xi \vdash \text{rec}(\alpha.A)}
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how do we define

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Should respect unfolding, e.g.,

$$
[\Xi \vdash \text{rec}(\alpha.A)] \cong [\Xi \vdash [\text{rec}(\alpha.A)/\alpha]A]
= [\Xi, \alpha \vdash A] (\neg, [\Xi \vdash \text{rec}(\alpha.A)])
$$
Given $F : \mathbf{DCPO}^{\Xi,\alpha} \to \mathbf{DCPO}$, we can find a parametrized family of fixed points $F^\dagger : \mathbf{DCPO}^{\Xi} \to \mathbf{DCPO}$ by taking:

$$F^\dagger D = \text{FIX}(F(D, -)).$$
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The family satisfies for all $D$:

$$F^\dagger D \simeq F(D, F^\dagger D)$$
Parametrized Families of Fixed Points

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**Example:** if $F = [\Xi, \alpha \vdash A]$ and $[\Xi \vdash \text{rec}(\alpha.A)] = F^\dagger$, then

$$[\Xi \vdash \text{rec}(\alpha.A)] \simeq [\Xi, \alpha \vdash A](-, [\Xi \vdash \text{rec}(\alpha.A)]).$$
Interpreting Recursive Session Types

Is this $(\cdot)^\dagger$ functorial? Does it preserve embeddings?
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**Wrong:** Take \((\exists \vdash \text{rec}(\alpha.A))\) to be

\[
(\exists, \alpha \vdash A)^\dagger : ([\exists, \alpha \vdash A]^\dagger \Rightarrow ([\exists, \alpha \vdash A] \times [\exists, \alpha \vdash A])^\dagger).
\]

**Problem:** What are \([\exists \vdash \text{rec}(\alpha.A)]\) and \([\exists \vdash \text{rec}(\alpha.A)]\)?

In general, \((F \times G)^\dagger \not\cong F^\dagger \times G^\dagger\).
We give a parametrized fixed-point operator $(\cdot)\dagger$ on $\mathcal{O}$-categories suitable for interpreting recursive session types. It is locally continuous and satisfies the Conway identities.
O-Categories and Local Continuity

Definition

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Definition

An **embedding-projection pair** (e-p-pair) is a pair of morphisms $e : A \to B$ and $p : B \to A$ such that $p \circ e = \text{id}_A$ and $e \circ p \sqsubseteq \text{id}_B$. 

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Output: Object $\text{FIX}(F)$ such that $F(\text{FIX}(F)) \simeq \text{FIX}(F)$
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Define: $\Omega(F) : \omega \to K$ by

$$\Omega(F)(n \to n + 1) = F^n ! : F^n \bot \to F^{n+1} \bot.$$
Canonical Fixed Points

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$$\Omega(F) : \bot \xrightarrow{\iota} F \bot$$
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$$\Omega(F) : \perp \xrightarrow{!} F \perp \xrightarrow{F!} F^2 \perp \xrightarrow{F^2!} F^3 \perp \xrightarrow{F^3!} \ldots$$
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Define: $\text{FIX}(F) = \text{colim } \Omega(F)$. 
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\[ \begin{array}{cccccc}
\Omega(F) : & \bot & ! & F \bot & F! & F^2 \bot & F^2! & F^3 \bot & F^3! & \ldots \\
\text{colim} \Omega(F) & \leftarrow & \leftarrow & \leftarrow & \leftarrow & \leftarrow & \leftarrow & \leftarrow & \leftarrow & \\
\end{array} \]
Generalizing Links

Let $\text{Links}_K$ be:

**Objects:** $(C, c, F)$ where $F : K \xrightarrow{\text{l.c.}} K$ and $c : C \to FC$ is an embedding;
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**Morphisms:** \((h, \eta) : (C, k, F) \to (D, d, G)\) is \( h : C \to D \) and (natural) \( \eta : F \Rightarrow G \) satisfying

\[
\begin{array}{ccc}
C & \xrightarrow{c} & FC \\
\downarrow h & & \downarrow \eta_C \\
D & \xrightarrow{d} & GD
\end{array}
\]

\[
\begin{array}{ccc}
& Fh & \\
\eta_C \downarrow & & \eta_C \downarrow \\
& \eta_D & \\
\downarrow & & \downarrow Gh \\
FD & \xleftarrow{\eta_D} & GC
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\]
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\[
\begin{array}{ccc}
FD & \xrightarrow{\eta_D} & \eta_h \\
\downarrow \eta & & \downarrow \eta_D \\
GC & \xrightarrow{\eta_C} & GD
\end{array}
\]

Write \( \eta \ast h \) for \( \eta_D \circ Fh = Gh \circ \eta_C \).
Define $\Omega : \text{Links}_K \rightarrow [\omega \rightarrow K]$ by

$$
\Omega(C, c, F) : \quad C \xrightarrow{c} FC \xrightarrow{Fc} F^2 C \xrightarrow{F^2c} F^3 C \xrightarrow{F^3c} \ldots
$$

where $\eta(n) = \eta^* \cdots \eta^*$ for $\eta : F \Rightarrow G$. 
Define $\Omega : \text{Links}_K \rightarrow [\omega \rightarrow K]$ by

$$
\begin{align*}
\Omega(C, c, F) : & \quad C \xrightarrow{c} FC \xrightarrow{Fc} F^2C \xrightarrow{F^2c} F^3C \xrightarrow{F^3c} \ldots \\
\Omega(D, d, G) : & \quad D \xrightarrow{d} GD \xrightarrow{Gd} G^2D \xrightarrow{G^2d} G^3D \xrightarrow{G^3d} \ldots \\
\end{align*}
$$

where $\eta^{(n)} = \eta \ast \cdots \ast \eta : F^n \Rightarrow G^n$ for $\eta : F \Rightarrow G$. 
Proposition. $\text{GFIX} = \text{colim} \circ \Omega : \text{Links}_K \xrightarrow{\text{I.c.}} K$ is locally continuous.
**Proposition.** $\text{GFIX} = \text{colim} \circ \Omega : \text{Links}_K \xrightarrow{\text{l.c.}} K$ is locally continuous.

**Proposition.** $\text{UNF}(C, c, F) = F(\text{GFIX}(C, c, F))$ extends to a functor $\text{Links}_K \xrightarrow{\text{l.c.}} K$. 

General Fixed Points

**Proposition.** $\text{GFIX} = \text{colim} \circ \Omega : \text{Links}_K \xrightarrow{\text{l.c.}} K$ is locally continuous.

**Proposition.** $\text{UNF}(C, c, F) = F(\text{GFIX}(C, c, F))$ extends to a functor $\text{Links}_K \xrightarrow{\text{l.c.}} K$.

**Theorem.** There exists a natural isomorphism

$$\text{fold} : \text{UNF} \Rightarrow \text{GFIX},$$

i.e., for all $(h, \eta) : (C, c, F) \rightarrow (D, d, G),$

$$\begin{array}{c}
F(\text{GFIX}(C, c, F)) \xrightarrow{\text{fold}_{(C,c,F)}} \text{GFIX}(C, c, F) \\
\text{UNF}(h, \eta) \downarrow \hspace{2cm} \downarrow \text{GFIX}(h, \eta) \\
G(\text{GFIX}(D, d, G)) \xrightarrow{\text{fold}_{(D,d,G)}} \text{GFIX}(D, d, G).
\end{array}$$
Remark. The mapping $F \mapsto (\bot, !, F)$ embeds the category $[K \xrightarrow{\text{l.c.}} K]$ of locally continuous functors on $K$ into $\text{Links}_K$. 
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Corollary. The functor $(\_)^\dagger$ given by

$$[D \times K \xrightarrow{\text{l.c.}} K] \xrightarrow{\Lambda} [D \xrightarrow{\text{l.c.}} [K \xrightarrow{\text{l.c.}} K]] \xrightarrow{[\text{id}_D \rightarrow \text{GFIX}(\bot, !, -)]} [D \xrightarrow{\text{l.c.}} K]$$

is locally continuous.
Theorem. Let $F : D \times K \xrightarrow{\text{l.c.}} K$. There exists a natural isomorphism

$$\text{Fold}^F : F^\dagger \Rightarrow F \circ \langle \text{id}_D, F^\dagger \rangle$$

given by $\text{Fold}^F_D = \text{fold}_{(\bot,!, F(D, -))} : F^\dagger D \to F(D, F^\dagger D)$.

The definition of Fold is also natural in $F$. 
**Theorem.** Let \( F, H : D \times K \xrightarrow{\text{l.c.}} K \) and \( G, I : C \xrightarrow{\text{l.c.}} D \). Set \( F_G = F \circ (G \times \text{id}) \) and analogously for \( H_I \). Then

\[
F_G^\dagger = F^\dagger \circ G \\
\text{Fold}^{F_G} = \text{Fold}^F \circ G.
\]
**Theorem.** Let $F, H : D \times K \xrightarrow{1.c.} K$ and $G, I : C \xrightarrow{1.c.} D$. Set $F_G = F \circ (G \times \text{id})$ and analogously for $H_I$. Then

$$F_G^\dagger = F^\dagger \circ G$$

$$\text{Fold}^{F_G} = \text{Fold}^{F} G.$$

If $\phi : F \Rightarrow H$ and $\gamma : G \Rightarrow I$, then

$$(\phi \ast (\gamma \times \text{id}))^\dagger = \phi^\dagger \ast \gamma : F_G^\dagger \Rightarrow H_I^\dagger.$$
Let $F : D \times K \overset{\text{l.c.}}{\longrightarrow} K$.

**Definition.** An $F$-**algebra** is a pair $(G, \gamma)$ where $G : D \overset{\text{l.c.}}{\longrightarrow} K$ and $\gamma : F \circ \langle \text{id}, G \rangle \Rightarrow G$. 
Let $F : D \times K \xrightarrow{\text{l.c.}} K$.

**Definition.** An $F$-algebra is a pair $(G, \gamma)$ where $G : D \xrightarrow{\text{l.c.}} K$ and $\gamma : F \circ \langle \text{id}, G \rangle \Rightarrow G$.

**Definition.** An $F$-algebra homomorphism $(G, \gamma) \to (H, \eta)$ is a $\rho : G \Rightarrow H$ such that

\[
\begin{array}{ccc}
F \circ \langle \text{id}, G \rangle & \xrightarrow{\gamma} & G \\
\downarrow_{F \circ \langle \text{id}, \rho \rangle} & & \downarrow_{\rho} \\
F \circ \langle \text{id}, H \rangle & \xrightarrow{\eta} & H.
\end{array}
\]
Theorem. Let $F : D \times K \xrightarrow{\text{l.c.}} K$.

- $(F^\dagger, \text{Fold}^F)$ is the initial $F$-algebra. Given any other $F$-algebra $(G, \gamma)$, the unique morphism $\phi : F^\dagger \Rightarrow G$ is a natural family of embeddings.
- $(F^\dagger, (\text{Fold}^F)^{-1})$ is the terminal $F$-coalgebra. Given any other $F$-coalgebra $(G, \gamma)$, the unique morphism $\rho : G \Rightarrow F^\dagger$ is a natural family of projections.
The interpretation

\[ (\Xi \vdash \text{rec}(\alpha.A)) : [\Xi \vdash \text{rec}(\alpha.A)] \Rightarrow \\
[\Xi \vdash \text{rec}(\alpha.A)] \times [\Xi \vdash \text{rec}(\alpha.A)] \]

is given by

\[ \langle (\pi_1(\Xi, \alpha \vdash A))^\dagger, (\pi_2(\Xi, \alpha \vdash A))^\dagger \rangle : \\
[\Xi, \alpha \vdash A]^\dagger \Rightarrow [\Xi, \alpha \vdash A]^\dagger \times [\Xi, \alpha \vdash A]^\dagger. \]

It is a natural family of embeddings by the theorem on the previous slide.
The **Conway identities** are four identities for dagger operations useful for semantic reasoning. They include:

1. the **parameter identity** (naturality):
   for all \( f : B \times C \rightarrow C \) and \( g : A \rightarrow B \),
   \[
   (f \circ (g \times \text{id}_C))^\dagger = f^\dagger \circ g.
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2. **the composition identity (parametrized dinaturality):**
   For all $f : P \times A \to B$ and $g : P \times B \to A$,
   \[
   (g \circ \langle \pi_P^{P \times A}, f \rangle)^{\dagger} = g \circ \langle \text{id}_P, (f \circ \langle \pi_P^{P \times B}, g \rangle)^{\dagger} \rangle. 
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Conway Identities

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1. the parameter identity (naturality):
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2. the composition identity (parametrized dinaturality):
   for all \( f : P \times A \to B \) and \( g : P \times B \to A \),
   \[
   (g \circ \langle \pi_{P \times A}^P, f \rangle)^\dagger = g \circ \langle \text{id}_P, (f \circ \langle \pi_{P \times B}^P, g \rangle)^\dagger \rangle.
   \]

Theorem. The dagger \((\cdot)^\dagger\) satisfies the Conway identities.
The Conway identities imply:

**Corollary (Pairing / Bekič’s Identity).** Let $F : A \times B \times C \xrightarrow{l.c.} B$ and $G : A \times B \times C \xrightarrow{l.c.} C$. Set

$$H = A \times B \xrightarrow{\langle \text{id}, G^\dagger \rangle} A \times B \times C \xrightarrow{F} B.$$ 

Then

$$\langle F, G \rangle^\dagger = \langle G^\dagger \circ \langle \text{id}_A, H^\dagger \rangle, H^\dagger \rangle : A \to B \times C.$$
The Conway identities imply:

**Corollary (Pairing / Bekič’s Identity).** Let \( F : A \times B \times C \xrightarrow{\text{l.c.}} B \) and \( G : A \times B \times C \xrightarrow{\text{l.c.}} C \). Set

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H = A \times B \xrightarrow{\langle \text{id}, G^\dagger \rangle} A \times B \times C \xrightarrow{F} B.
\]

Then

\[
\langle F, G \rangle^\dagger = \langle G^\dagger \circ \langle \text{id}_A, H^\dagger \rangle, H^\dagger \rangle : A \to B \times C.
\]

**Application.** Interpreting and reasoning about mutually recursive session types.
Applications to Session Types

These techniques are key to a domain semantics for session-typed languages with general recursion.
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**Benefit:** compositional semantics and program equivalence.
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2. process composition is associative
Applications to Session Types

These techniques are key to a domain semantics for session-typed languages with general recursion.

**Benefit:** compositional semantics and program equivalence.

Has been used to verify, e.g., that:

1. flipping bits in a bit stream twice is the identity
2. process composition is associative
3. large class of $\eta$-like properties
We gave a parametrized fixed-point operator \((\cdot)^\dagger\) that is:

- **locally continuous**;
- satisfies the **Conway identities**;
- useful for interpreting **recursive session types**.
Related Work


Lehmann, Daniel J. and Michael B. Smyth. 
**Algebraic Specification of Data Types: A Synthetic Approach**

**The Category-Theoretic Solution of Recursive Domain Equations**